Suggested Solutions to:
Regular Exam, June 3, 2021
Industrial Organization

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Question 1: Strategic delegation

Part (a)

The game consists of two stages. At the first stage the owners, independently and simultaneously, choose an instruction P_i or R_i. At the second stage we have four different possibilities, depending on what instructions the owners have chosen: both firms are profit maximizers, (P_1, P_2); both firms are revenue maximizers, (R_1, R_2); or one is a profit maximizer and the other is a revenue maximizer, (P_1, R_2) or (R_1, P_2). Given these objectives, the managers choose, independently and simultaneously, a quantity q_i.

We can solve for the subgame-perfect Nash equilibria of the model by backward induction. We therefore start by solving the four second-stage subgames.

- **The case (P_1, P_2).** Each firm maximizes

$$[45 - 9(q_1 + q_2)]\, q_i - 9q_i = [36 - 9(q_1 + q_2)]\, q_i.$$

The first-order conditions for the two firms are

$$-9q_1 + [36 - 9(q_1 + q_2)] = 0$$

and

$$-9q_2 + [36 - 9(q_1 + q_2)] = 0.$$

Solving these equations for q_1 and q_2 yields

$$\left(q_1^{PP}, q_2^{PP}\right) = \left(\frac{4}{3}, \frac{4}{3}\right).$$

The profit levels given these outputs are

$$\pi_1^{PP} = \left[45 - 9\left(q_1^{PP} + q_2^{PP}\right)\right] q_1^{PP} - 9q_1^{PP} = 16$$

and

$$\pi_2^{PP} = \left[45 - 9\left(q_1^{PP} + q_2^{PP}\right)\right] q_2^{PP} - 9q_2^{PP} = 16.$$

- **The case (R_1, R_2).** Each firm maximizes its revenues

$$[45 - 9(q_1 + q_2)]\, q_i.$$

The first-order conditions for the two firms are

$$-9q_1 + [45 - 9(q_1 + q_2)] = 0$$

and

$$-9q_2 + [45 - 9(q_1 + q_2)] = 0.$$

Solving these equations for q_1 and q_2 yields

$$\left(q_1^{RR}, q_2^{RR}\right) = \left(\frac{5}{3}, \frac{5}{3}\right).$$

The profit levels given these outputs are

$$\pi_1^{RR} = \left[45 - 9\left(q_1^{RR} + q_2^{RR}\right)\right] q_1^{RR} - 9q_1^{RR} = 10$$

and

$$\pi_2^{RR} = \left[45 - 9\left(q_1^{RR} + q_2^{RR}\right)\right] q_2^{RR} - 9q_2^{RR} = 10.$$

- **The case (P_1, R_2).** Firm 1 maximizes its profit

$$\left[45 - 9\left(q_1 + q_2\right)\right] q_i - 9q_i = \left[36 - 9\left(q_1 + q_2\right)\right] q_i.$$

Firm 1's first-order condition is

$$-9q_1 + \left[36 - 9\left(q_1 + q_2\right)\right] = 0. \tag{1}$$

Firm 2 maximizes its revenues

$$\left[45 - 9\left(q_1 + q_2\right)\right] q_i.$$

Firm 2's first-order condition is

$$-9q_2 + \left[45 - 9\left(q_1 + q_2\right)\right] = 0. \tag{2}$$

Solving equations (1) and (2) for q_1 and q_2 yields

$$\left(q_1^{PR}, q_2^{PR}\right) = (1, 2).$$

The profit levels given these outputs are

$$\pi_1^{PR} = \left[45 - 9\left(q_1^{PR} + q_2^{PR}\right)\right] q_1^{PR} - 9q_1^{PR} = 9$$

and

$$\pi_2^{PR} = \left[45 - 9\left(q_1^{PR} + q_2^{PR}\right)\right] q_2^{PR} - 9q_2^{PR} = 18.$$

- **The case (R_1, P_2).** This is symmetric relative to the case (P_1, R_2). Therefore, $\left(q_1^{RP}, q_2^{RP}\right) = (2, 1)$,

$$\pi_1^{RP} = 18,$$

and

$$\pi_2^{RP} = 9.$$

- We have now solved all the stage 2 subgames and derived expressions for the equilibrium profit levels in all of these. Using these profit levels we can illustrate the stage 1 interaction between O_1 and O_2 in a game matrix (where O_1 is the row player and O_2 is the column player):

	P_2	R_2
P_1	16, 16	9, 18
R_1	18, 9	10, 10

We see that each player has a strictly dominant strategy and that, in particular, the unique Nash equilibrium of the stage 1 game is that both owners choose revenue maximization, (R_1, R_2).

- **Conclusion:** the game has a unique subgame perfect Nash equilibrium. In this equilibrium, both owners choose revenue maximization, (R_1, R_2). In the stage 2 equilibrium path subgame, the managers choose $\left(q_1^{RR}, q_2^{RR}\right) = \left(\frac{5}{3}, \frac{5}{3}\right)$. In the three off-the-equilibrium path subgames, the managers choose $\left(q_1^{PP}, q_2^{PP}\right) = \left(\frac{4}{3}, \frac{4}{3}\right)$, $\left(q_1^{PR}, q_2^{PR}\right) = (1, 2)$, and $\left(q_1^{RP}, q_2^{RP}\right) = (2, 1)$.

Part (b)

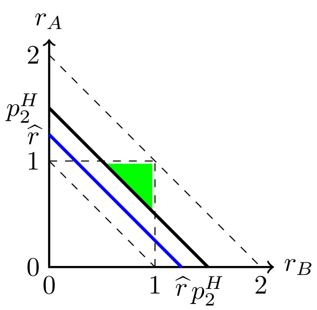
Interpretation: The owners would be better off if they both chose to instruct their manager to maximize profit. The reason why this cannot be part of an equilibrium is that each firm can gain by unilaterally instruct its own manager to maximize revenues instead. Why is this the case? First, a manager who maximizes revenues will be more aggressive (i.e., produce more) than a profit maximizing manager. Second, the rival manager, expecting this behavior, will respond by producing less (since the firms' outputs are strategic substitutes). This will increase the first firm's market share and profit.

- If the managers' choice variables had been strategic complements instead we should expect the opposite result: each firm would like to make the rival behave in a way that is good for the own profits (i.e., charge a high price or choose a small quantity). If the choice variables are strategic complements, this means that to induce the rival to behave like that a firm should behave in the same way itself (i.e., charge a high price or choose a small quantity). Therefore, an owner could gain by instructing its manager to be relatively non-aggressive (i.e., to have a strong incentive to charge a high price or choose a small quantity)—this can be achieved by instructing the manager to maximize profits rather than revenues.
- The assumption that the instruction is observable by the rival firm is crucial. Without that assumption, an owner would always want the own manager to maximize profits (but maybe still be *telling* the rival manager that the instruction was R). The point with choosing R is that then the rival *knows* this (and knows that this choice is irreversible), which will (in the model with strategic substitutes) have a beneficial effect on the rival manager's optimal choice at the second stage.

Question 2: Behavior-based price discrimination and bundling

(a) Solve the firm's profit-maximization problem in the second-period H market. That is, derive an expression for p_2^H, as a function of $\widehat{r}$, in an equilibrium where the conditions in (2) hold (assuming that such an equilibrium exists).

We first need to derive demand in the second-period H market. To this end, note that the consumers that belong to the H market are those for whom $r_A + r_B \geq \widehat{r}$ (or, rewriting, $r_A \geq \widehat{r} - r_B$). Thus, the H market consumers are those with (r_A, r_B) values in the unit square and on, or north-east of, the blue straight line in the figure below.[1] Among these consumers, those with $r_A + r_B \geq p_2^H \Leftrightarrow r_A \geq p_2^H - r_B$ will purchase the bundle, which means that demand in the H market is given by the size of the green area in the figure.



The green area is a triangle with a base and a height that both equal $1 - (p_2^H - 1) = 2 - p_2^H$. Hence, the green area equals $(2 - p_2^H)^2/2$, and demand in the H market is given by

$$Q_2^H = \begin{cases} \frac{(2-p_2^H)^2}{2} & \text{if } p_2^H \geq \widehat{r}; \\ \frac{(2-\widehat{r})^2}{2} & \text{if } p_2^H < \widehat{r}. \end{cases} \tag{3}$$

The firm's profit in the H market can be written as $\pi_2^H = Q_2^H p_2^H$. This function is strictly increasing in p_2^H for all $p_2^H < \widehat{r}$ (cf. the second row of (3)). For $p_2^H \geq \widehat{r}$, we can compute the following marginal profit expression:

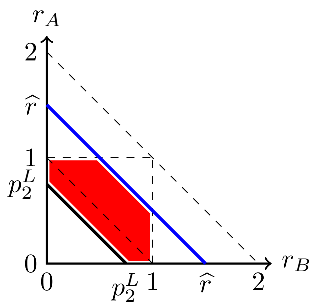
$$\frac{\partial \pi_2^H}{\partial p_2^H} = -(2 - p_2^H)p_2^H + \frac{(2 - p_2^H)^2}{2} = \frac{(2 - p_2^H)(2 - 3p_2^H)}{2} < 0, \tag{4}$$

where the last inequality must hold as $p_2^H \geq \widehat{r} \geq 1 > \frac{2}{3}$. That is, the profit function is increasing for $p_2^H < \widehat{r}$, and it is decreasing for $p_2^H \geq \widehat{r}$. It follows that the firm's optimal price in the second-period H market is given by $p_2^H = \widehat{r}$.

[1] The figure is consistent with $\widehat{r} \geq 1$, which we were supposed to assume. However, the condition $\widehat{r} \geq p_2^H$ is violated in the figure. Still, the reasoning below will tell us what demand is both for the case $\widehat{r} < p_2^H$ and the case $\widehat{r} \geq p_2^H$.

(b) Solve the firm's profit-maximization problem in the second-period L market. That is, derive an expression for p_2^L, as a function of $\widehat{r}$, in an equilibrium where the conditions in (2) hold (assuming that such an equilibrium exists). You should assume that the firm's optimal value of p_2^L satisfies $p_2^L \le 1$.

We first need to derive demand in the second-period L market. To this end, note that the consumers that belong to the L market are those for whom $r_A + r_B \le \widehat{r}$ (or, rewriting, $r_A \le \widehat{r} - r_B$). Thus, the L market consumers are those with (r_A, r_B) values in the unit square and on, or south-west of, the blue straight line in the figure below. Among these consumers, those with $r_A + r_B \ge p_2^L \Leftrightarrow r_A \ge p_2^L - r_B$ will purchase the bundle, which means that demand in the L market is given by the size of the red area in the figure (the figure is consistent with $p_2^L \le 1$, which we were supposed to assume).



The red area can be computed as one minus the triangle above it and minus the triangle below it. The triangle above the red area represents the consumers who belong to the H market, and its size is $(2-\widehat{r})^2/2$. The triangle below the red area represents the L market consumers who choose not to buy, and its size is $(p_2^L)^2/2$. Hence, demand in the L market is given by

$$Q_2^L = 1 - \frac{(2-\widehat{r})^2}{2} - \frac{(p_2^L)^2}{2} \tag{5}$$

The firm's profit in the L market can thus be written as

$$\pi_2^L = Q_2^L p_2^L = \left[1 - \frac{(2-\widehat{r})^2}{2} - \frac{(p_2^L)^2}{2}\right] p_2^L. \tag{6}$$

The first-order condition is given by

$$\frac{\partial \pi_2^L}{\partial p_2^L} = 1 - \frac{(2-\widehat{r})^2}{2} - \frac{3(p_2^L)^2}{2} = 0. \tag{7}$$

The second order condition is $\frac{\partial^2 \pi_2^L}{\partial (p_2^L)^2} = -3p_2^L < 0$, which clearly is satisfied for all $p_2^L \in (0,1]$. Hence, the optimal price p_2^L can be characterized by the first-order condition above, which can be rewritten as

$$p_2^L = \sqrt{\frac{2}{3}\left[1 - \frac{(2-\widehat{r})^2}{2}\right]}. \tag{8}$$

(c) Does there exist an equilibrium of the model where the conditions in (2) hold? Show that such an equilibrium exists, or show that such an equilibrium does not exist.

An equilibrium of the model with $\widehat{r} \ge 1$ does *not* exist. To see this, we can solve the firm's profit-maximization problem in period 1, given that it expects the second prices that we derived under (a) and (b) to be played, and given the relationship between p_1 and $\widehat{r}$ that must hold at an equilibrium.

The firm's first-period profits can be written as $\pi_1 = p_1 Q_1$, where $p_1 Q_1$ is the firm's first-period demand. By definition of $\widehat{r}$, the first-period demand equals the mass of consumers for whom $r_A + r_B \ge \widehat{r}$. Hence, from the arguments under (a), we have $Q_1 = \frac{(2-\widehat{r})^2}{2}$. Thus,

$$\pi_1 = p_1 \frac{(2-\widehat{r})^2}{2} \tag{9}$$

The relationship between p_1 and $\widehat{r}$ can be obtained by changing inequality (1) in the exam paper to an equality, and then evaluating at $r_A + r_B = \widehat{r}$ and at the optimal second-period prices obtained under (a) and (b). Doing those things yields

$$\widehat{r} - p_1 + (\widehat{r} - r_2^H) = \widehat{r} - p_2^L \Leftrightarrow p_1 = p_2^L = \sqrt{\frac{2}{3}\left[1 - \frac{(2-\widehat{r})^2}{2}\right]}. \tag{10}$$

The first-period profits are thus given by

$$\pi_1 = \left[\frac{2(1-x)}{3}\right]^{\frac{1}{2}} x, \qquad \text{where} \quad x \stackrel{\text{def}}{=} \frac{(2-\widehat{r})^2}{2}. \tag{11}$$

It is straightforward to verify that π_1 is maximized at $x = \frac{2}{3}$, which means that (using the relationship between x and $\widehat{r}$ stated in (11))

$$\widehat{r} = 2 - \sqrt{\frac{4}{3}} < 1.$$

That is, we have found that if the players of the game expect $\widehat{r} \geq 1$, then they have an incentive to behave in a way that gives rise to $\widehat{r} < 1$, which is inconsistent with their beliefs. This means that there is no equilibrium with $\widehat{r} \geq 1$.

(d) Suppose we solved for an equilibrium of the model described above (not necessarily one where (2) holds). Suppose that we also solved for the equilibrium of a variation of that model where the firm, in both periods, sells the two goods separately (but all other assumptions are the same). In which one of the two models should we expect the firm to earn the highest equilibrium profits—the model with bundling or the model with separate prices?

- You do not need to answer with a definite "bundling" or a definite "separate prices." Instead, discuss different reasons—perhaps by referring to related models that we have studied in the course—for why we should expect either the first model or the second model to yield the highest equilibrium profits.

It seems hard to see for sure, only by verbal reasoning, which one of the models that would yield the highest profits. Probably one would have to solve the two models fully and then make the comparison algebraically. However, a few different reasons for why we should expect the comparison to go in one of the two directions are discussed below (perhaps the students can add further arguments).

- We studied a similar model of bundling in the course. There the monopoly firm was selling its two goods in a single period and, hence, behavior-based price discrimination was not an issue. However, in all other ways, the model assumptions were consistent with the ones here (for example, the distribution of consumers was uniform).

  In that related model in the course, we found that bundling was profitable. This suggests that a similar effect is likely to be present also in the two-period model studied here. Thus, all else equal, this argument suggests that the profits are higher under bundling.

- We know that a monopoly firm's opportunity to practice price discrimination (at least in a static setting—cf. the discussion under next bullet point) cannot hurt it, as the firm can always choose to replicate the price it would have chosen under a ban on price discrimination. Indeed, we should expect that, on some occasions, price discrimination is strictly beneficial for a monopoly firm. Moreover, if a firm sells the goods separately (at least in the first period), then it can in the second period condition its price on a larger number of consumer behaviors (namely, whether a consumer in the first period purchased both A and B, only A, only B, or neither good). If more opportunities to price discriminate yield higher profits, this argument suggests (all else being equal) that the firm prefers to sell the goods separately.

- In the model of behavior-based price discrimination that we studied in the course, there was a so-called Coase conjecture effect present: the fact that buying in period 1 leads to a higher second period price than otherwise makes the consumer reluctant to purchase in the first period; to encourage the consumers to buy also in the first period, the firm must lower the first period price and it loses market power (the logic is similar to the one in a durable goods monopoly market). As a consequence, in that environment the firm would benefit from committing itself not to practice price discrimination in period 2.

In the present model with two goods (and bundling), we should still expect a Coase conjecture effect to be present. It is not obvious if this effect is strongest under bundling or separate prices. However, a plausible guess would be that—since separate prices creates more opportunity for price discrimination (which is the root of the firm's problem with loss of market power)—the Coase conjecture effect is strongest with separate prices. If so, we should conclude that the firm prefers to choose bundling.

A more cautious conclusion would be that the Coase conjecture effect most likely matters for the question whether the firm prefers bundling or separate prices, but that it is hard to see in what way.